

STATUS OF THE CKM MATRIX

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The experimental status and theoretical uncertainties of the Cabibbo–Kobayashi–Maskawa (CKM) matrix describing the charge-changing weak transitions between quarks with charges $-1/3$ (d, s, b) and $2/3$ (u, c, t) are reviewed. Some recent methods of obtaining phases of CKM elements are described.

1 Introduction

Information about the Cabibbo–Kobayashi–Maskawa (CKM) matrix describing the charge-changing weak transitions between quarks with charges $-1/3$ (d, s, b) and $2/3$ (u, c, t) has been steadily improving over the years. Despite a wealth of overconstraining experiments, no significant inconsistencies in its parameters have emerged so far. One seeks greater accuracy in the determination of CKM elements not only to expose such inconsistencies, which could signal new physics, but also to provide input for an eventual theory of these elements.

The matrix may be defined in one parametrization¹ as

$$V_{\text{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \simeq \begin{bmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{bmatrix}. \quad (1)$$

Here $\bar{\rho} \equiv \rho(1 - \lambda^2/2)$, $\bar{\eta} \equiv \eta(1 - \lambda^2/2)$, with $\lambda \simeq 0.225$, $A \simeq 0.8$, $\bar{\eta} \simeq 0.36$, $\bar{\rho} \simeq 0.19$. Many detailed reviews exist^{2,3,4}; we concentrate on procedures and open questions.

We shall be concerned with information regarding both magnitudes and phases of CKM elements. These are encoded in the angles of the *unitarity triangle*, illustrated in Fig. 1. Current fits (not including some CP asymmetries providing information on α and γ) imply 1σ limits⁵

$$\beta = (23.8^{+2.1}_{-2.0})^\circ, \quad \alpha = (94^{+12}_{-10})^\circ, \quad \gamma = (62^{+10}_{-12})^\circ. \quad (2)$$

Thus, although β is well known [with the ICHEP 2004 average now $(23.3^{+1.6}_{-1.5})^\circ$]⁶, α and γ are more uncertain, ranging over about 40° at the 95% confidence level (c.l.). They can be pinned down more precisely using B – \bar{B} mixing, kaon decays, and CP asymmetries in B decays.

2 V_{ud} from nuclear, neutron, pion β decays

Our discussion is based on a recent overview⁷. Nine measurements of nuclear $0^+ \rightarrow 0^+$ transitions yield an average of $|V_{ud}| = 0.9740(1)(3)(4)$, where the errors correspond to experiment, nuclear theory, and radiative corrections, respectively. Neutron decay gives $|V_{ud}|^2(1 + 3g_A^2)\tau_n = (4908 \pm 4)$ s, so using the measured lifetime $\tau_n = 885.7(7)$ s and $g_A = 1.2720(18)$ one finds $|V_{ud}| = 0.9729(4)(11)(4)$, where the errors are associated with τ_n , g_A , and radiative corrections. (A very new value⁸ $\tau_n = 878.5(7)(3)$ s implies $|V_{ud}| = 0.9757(4)(11)(4)$.) Pion beta decay ($\pi^+ \rightarrow \pi^0 e^+ \nu_e$) yields $|V_{ud}| = 0.9739(39)$; an ongoing experiment at PSI⁹ seeks to reduce the errors further. The overall average (before including the result of Ref.⁸) is $|V_{ud}| = 0.9740(5)$.

3 V_{us} : Hyperon and $K_{\ell 3}$ decays; lattice

Semileptonic hyperon decays, including new measurements of $\Lambda \rightarrow pe^- \bar{\nu}$, $\Sigma^- \rightarrow ne^- \bar{\nu}$, $\Xi^- \rightarrow \Lambda e^- \bar{\nu}$, and $\Xi^0 \rightarrow \Sigma^+ e^- \bar{\nu}$, have been analyzed¹⁰, with the result $|V_{us}| = 0.2250 \pm 0.0027$. Sat-

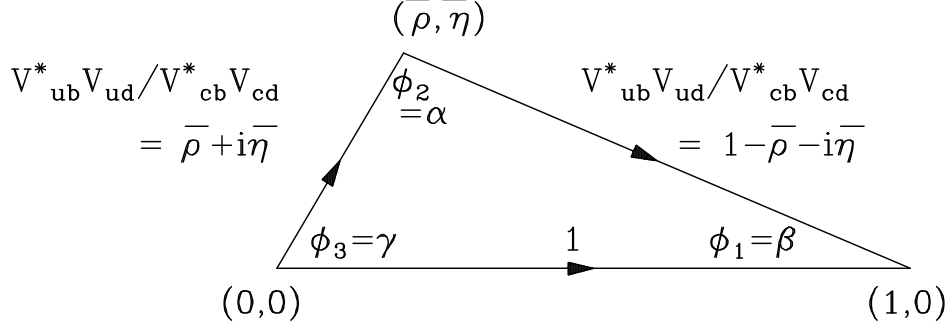


Figure 1: Definition of sides and angles of the unitarity triangle.

isfactory fits to all data have been found without the need for SU(3) breaking, though another analysis¹¹ requires it, obtaining $|V_{us}| = 0.2199 \pm 0.0026$. One remaining question is the magnitude of the axial weak-magnetism parameter g_2 , which is not well constrained by data.

Several experiments have remeasured $K_{\ell 3}$ decays. Brookhaven E865¹² finds $|V_{us}| = 0.2272 \pm 0.0022_{\text{rate}} \pm 0.0007_{\text{f.f.}} \pm 0.0018_{f_+(0)}$; radiative corrections were an important part of the analysis. Fermilab E832 (the KTeV Collaboration)¹³ obtains $|V_{us}| = 0.2252 \pm 0.0008_{\text{KTeV}} \pm 0.0021_{\text{ext}}$, where “ext” refers to all errors external to those in KTeV, such as uncertainties in form factors, making use of the radiative corrections in¹⁴.

The value of $|V_{ud}| = 0.9740 \pm 0.0005$, when combined with the Particle Data Group⁴ value $|V_{us}| = 0.2200 \pm 0.0026$ agreed poorly with CKM unitarity: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9971 \pm 0.0015$, while the expected value of $|V_{us}|$ from unitarity is $|V_{us}| = (1 - |V_{ud}|^2 - |V_{ub}|^2)^{1/2} = 0.2265 \pm 0.0023$. The above two $K_{\ell 3}$ results, as well as that¹⁰ from hyperon decays (see, however, Ref.¹¹), are much more consistent with unitarity. So, too, are very recently reported results from the KLOE detector at the DAΦNE e^+e^- collider at Frascati¹⁵. The NA48 Collaboration¹⁶ reports a value $|V_{us}| = 0.2187 \pm 0.0028$ using a value of $f_+(0)$ about 1.35% higher than that used by KTeV, so the situation is still not entirely settled.

4 V_{cd} and V_{cs} : Charm and W decays; neutrino production

The CLEO III detector has reported rates and spectra for $D^0 \rightarrow \pi^- \ell^+ \nu$ and $D^0 \rightarrow K^- \ell^+ \nu$ ¹⁷ implying $|f_+^\pi(0)|^2 |V_{cd}|^2 / |f_+^K(0)|^2 |V_{cs}|^2 = 0.038^{+0.006+0.005}_{-0.007-0.003}$. Progress in $n_f = 3$ lattice QCD by the Fermilab-MILC Collaboration¹⁸ has yielded form factors $f_+^{D \rightarrow \pi}(0) = 0.64(3)(5)$, $f_+^{D \rightarrow K}(0) = 0.73(3)(6)$, where the lattice errors are statistical and systematic, leading when combined with previous measurements of nonstrange and strange charm decays to $|V_{cd}| = 0.239(10)(19)(20)$, $|V_{cs}| = 0.969(39)(78)(24)$, where the final errors are experimental.

Charm production by neutrinos (signaled by dileptons) leads⁴ to $\overline{\mathcal{B}}(c \rightarrow \ell + X) |V_{cd}|^2 = (4.63 \pm 0.34) \times 10^{-3}$ so that with $\overline{\mathcal{B}}(c \rightarrow \ell + X) = (9.23 \pm 0.73)\%$ one has $|V_{cd}| = 0.224 \pm 0.012$. New measurements by the CLEO-c Collaboration of $\mathcal{B}(D^0 \rightarrow [K^-, \pi^-] e^+ \nu)$ and $\mathcal{B}(D^+ \rightarrow [K^{*0}, \rho^0] e^+ \nu)$ with 57 pb^{-1} at the $\psi''(3770)$ ¹⁹ represent a further source of information on V_{cd} and V_{cs} when combined with the lattice results²⁰. For example, the CLEO-c result for $\mathcal{B}(D^0 \rightarrow \pi e \nu) / \mathcal{B}(D^0 \rightarrow K e \nu)$ is $0.070 \pm 0.007 \pm 0.003$, to be compared with CLEO-III’s $0.082 \pm 0.006 \pm 0.005$. Higher-luminosity running in $e^+e^- \rightarrow \psi''(3770) \rightarrow D\bar{D}$ is to begin in September, and further improvements are envisioned. The CLEO-c Collaboration eventually hopes for 3 fb^{-1} at the ψ'' . Lattice errors will be the limiting factor in extracting $|V_{cd}|$ to $\mathcal{O}(1\%)$.

Charm-tagged W decays at LEP II (ALEPH, DELPHI)⁴ have given the value $|V_{cs}| = 0.97 \pm 0.09 \pm 0.07$. Measurement of the leptonic branching ratio $\mathcal{B}(W \rightarrow \ell \nu)$ and the assumption of a

standard pattern of W decays can be used to improve this estimate through the relation

$$\frac{1}{\mathcal{B}(W \rightarrow \ell \nu)} = 3 \left(1 + \left[1 + \frac{\alpha_s(M_W)}{\pi} \right] \sum |V_{ij}|^2 \right) \quad (i = u, c; j = d, s, b) \quad (3)$$

which implies $\sum |V_{ij}|^2 = 2.039 \pm 0.025$ and hence $|V_{cs}| = 0.996 \pm 0.013$ when contributions of other CKM elements are subtracted. The study of Cabibbo-favored ($c \rightarrow s$) decays of charmed particles at CLEO-c will provide $|V_{cs}|$ at $\mathcal{O}(1\%)$ accuracy if the \mathcal{L} goal is achieved and if lattice gauge theory continues to progress.

At this juncture one can't see a violation of two-family unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 = |V_{cd}|^2 + |V_{cs}|^2 = |V_{ud}|^2 + |V_{cd}|^2 = |V_{us}|^2 + |V_{cs}|^2 = 1. \quad (4)$$

This reflects on the very hierarchical structure of the CKM matrix. It implies that violations of unitarity may be too small to signal the presence of additional quark families.

5 V_{cb} : $b \rightarrow c$ inclusive and exclusive decays

At the quark level, the $b \rightarrow c$ semileptonic decay rate is simple (neglecting m_ℓ):

$$\Gamma(b \rightarrow c \bar{\nu}_\ell \ell^-) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 f \left(\frac{m_c^2}{m_b^2} \right), \quad f(x) \equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x. \quad (5)$$

However, initial and final states contain real hadrons, so one must employ perturbative QCD; expansions in inverse heavy quark mass (“HQE”); and moments in lepton energy, hadron mass, and photon energy in $b \rightarrow s \gamma$ (see, e.g.,²¹ for a review of these techniques) to infer the inclusive value $|V_{cb}| = 0.0421 \pm 0.0013$ ²², which is quoted in⁵ as $0.0420 \pm 0.0006_{\text{stat}} \pm 0.0008_{\text{theo}}$. More recent contributions include $|V_{cb}| = (41.4 \pm 0.4_{\text{stat}} \pm 0.4_{\text{HQE}} \pm 0.6_{\text{theo}}) \times 10^{-3}$ ²³ and $|V_{cb}| = (42.4 \pm 0.8_{\text{stat+HQE}} \pm (> 0.8)_{\text{theo}}) \times 10^{-3}$ ²⁴.

The best source of $|V_{cb}|$ from exclusive decays is the process $B \rightarrow D^* \ell \bar{\nu}_\ell$, whose rate is

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} |V_{cb}|^2 (m_B - m_{D^*})^2 m_{D^*}^3 \sqrt{w^2 - 1} \mathcal{G}(w) |\mathcal{F}(w)|^2. \quad (6)$$

One Isgur-Wise form factor $\mathcal{F}(w) \simeq \mathcal{F}(1)[1 + \rho^2(w-1) \dots]$, a function of the variable $w = v_B \cdot v_{D^*}$, governs the process; phase space is described by a function $\mathcal{G}(w)$ with $\mathcal{G}(1) = 1$. The form factor at $w = 1$ is given by $\mathcal{F}(1) = \eta_{QCD}[1 + \mathcal{O}(1/m_b^2)] = 0.913_{-0.035}^{+0.030}$ as calculated using lattice and HQET estimates²⁵.

A compilation of values of $\mathcal{F}(1)|V_{cb}|^3$ is shown in Fig. 2. The latest average is $\mathcal{F}(1)|V_{cb}|^2 = (37.7 \pm 0.9) \times 10^{-3}$, with the form factor slope parameter $\rho^2 = 1.56 \pm 0.14$, leading to $|V_{cb}| = (41.4 \pm 1.0_{\text{exp}} \pm 1.8_{\text{th}}) \times 10^{-3}$. This form factor shape is consistent with the rates for $B^0 \rightarrow D^{(*)-} \pi^+$, $D^{(*)-} D_s^{(*)+}$ estimated with a factorization approach²⁶. It is consistent with the inclusive $|V_{cb}|$ value. Although it currently has larger errors, progress in experiment and lattice estimates may eventually make this the best source of $|V_{cb}|$.

6 V_{ub} : $B \rightarrow u$ inclusive and exclusive decays

The semileptonic decay $b \rightarrow u \bar{\nu}_\ell \ell^-$ is the source of information on V_{ub} from inclusive decays. However, $\Gamma(b \rightarrow u \bar{\nu}_\ell \ell^-)$ is only about 2% of $\Gamma(b \rightarrow c \bar{\nu}_\ell \ell^-)$. Several strategies have been used to extract the $b \rightarrow u \bar{\nu}_\ell \ell^-$ contribution²¹, including measurement of leptons beyond the $b \rightarrow c \bar{\nu}_\ell \ell^-$ end point, reconstruction of hadronic masses $M_X < M_D$, cutting on $q^2 = m_{\ell \nu}^2$, and, most

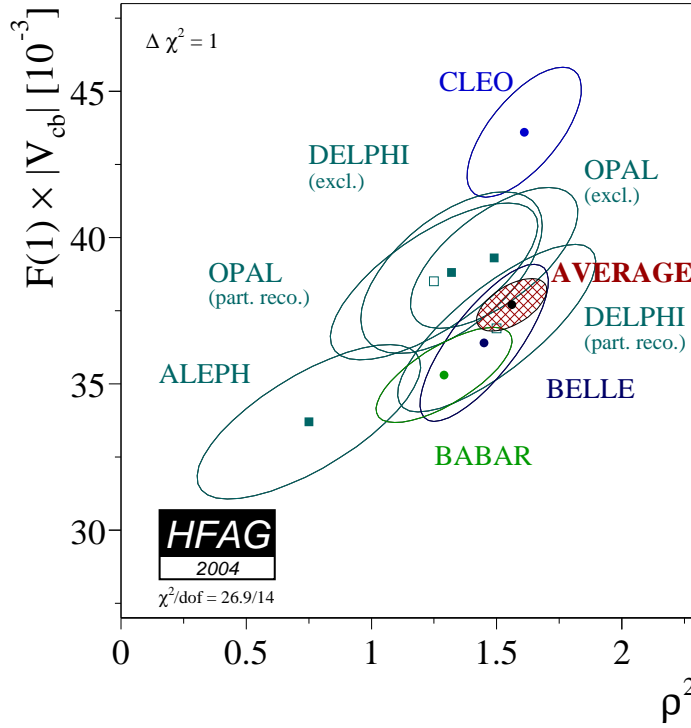


Figure 2: Values of $\mathcal{F}(1)|V_{cb}|$ vs. form factor parameter ρ^2 measured by various groups.

recently, light-cone-inspired kinematic cuts²⁷. As in $b \rightarrow c\bar{\nu}_\ell\ell^-$, the $b \rightarrow s\gamma$ photon spectrum helps to pin down hadronic uncertainties.

A compilation of values of $|V_{ub}|$ ³ is shown in Fig. 3. The bottom 5 points lead to an average $|V_{ub}| = (4.66 \pm 0.43) \times 10^{-3}$. This is notably higher than the value from exclusive $b \rightarrow u$ decays (e.g., $B \rightarrow \pi\bar{\nu}_\ell\ell$). The top 6 plotted points give an average of $|V_{ub}| = (3.26 \pm 0.62) \times 10^{-3}$ from exclusive decays. In this average I assigned an overall systematic error of ± 0.60 . Combining with the inclusive value and including a scale factor⁴ $S = 1.86 = \sqrt{\chi^2}$, I find $|V_{ub}| = (4.21 \pm 0.66) \times 10^{-3}$. The slight discrepancy between inclusive and exclusive values merits caution; quark-hadron duality may not be valid if extended down to hadronic masses which are so low as to be represented by discrete states like π and ρ .

The measurement of the spectrum for $B^0 \rightarrow \pi^- l^+ \nu_l$, e.g., as performed by CLEO²⁸, can be useful not only in extracting $|V_{ub}|$ from lattice gauge theory form factor results (typically obtained for high q^2), but also in testing factorization in $B^0 \rightarrow \pi^+ \pi^-$ ²⁹.

7 V_{td} : B^0 - \bar{B}^0 mixing; progress on decay constants

Loop diagrams with quarks $i, j = u, c, t$ in the intermediate state allow $b\bar{d} \leftrightarrow d\bar{b}$ transitions at 2nd order in weak interactions. The t quark dominates, so this mixing provides information on $|V_{td}|$. The predicted splitting Δm_d between mass eigenstates in the B^0 - \bar{B}^0 system is proportional to a parameter f_B^2 (the B meson decay constant) governing the matrix element of the $b\bar{d} \leftrightarrow d\bar{b}$ operator between physical meson states, and to a parameter B_B equal to 1 if W exchange diagrams dominate. A lattice estimate $f_B\sqrt{B_B} = (228 \pm 30 \pm 10)$ MeV and the experimental value $\Delta m_d \simeq 0.5$ ps⁻¹ imply the 95% c.l. range⁵ $|V_{td}| = (8.26_{-1.79}^{+1.23}) \times 10^{-3}$, equivalent to $|1 - \bar{\rho} - i\bar{\eta}| = 0.89_{-0.20}^{+0.12}$. (The upper limit on these quantities is governed by a lower limit on B_s - \bar{B}_s mixing to be discussed below.) For comparison, the 95% c.l. range⁵ of $|V_{ub}| = (3.87_{-0.61}^{+0.73}) \times 10^{-3}$

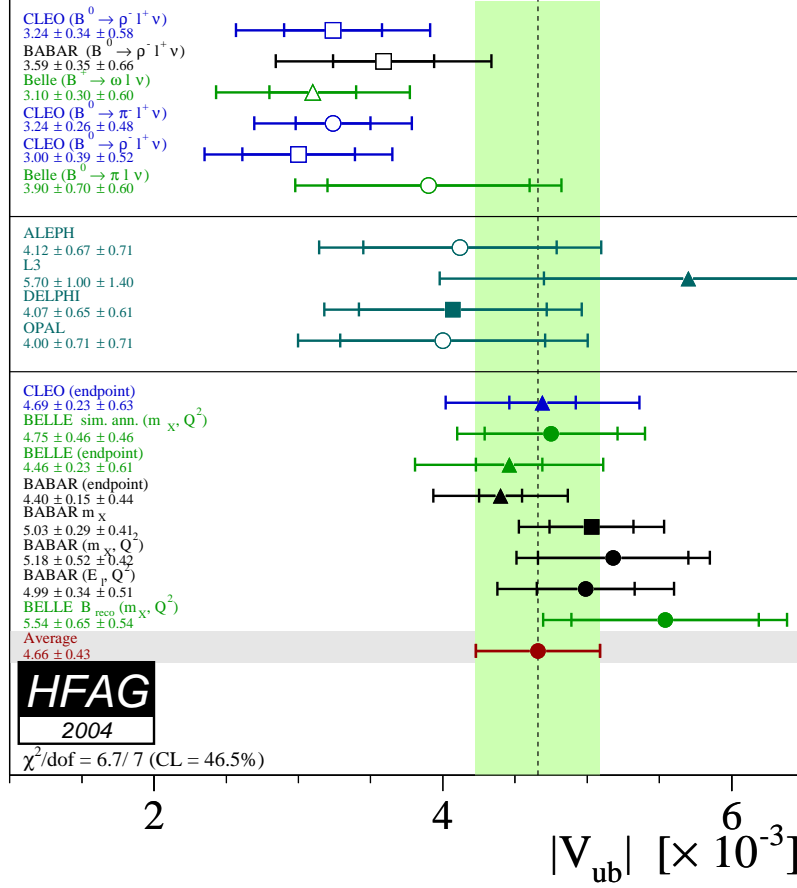


Figure 3: Values of $|V_{ub}|$ measured by various groups.

implies $|\bar{\rho} + i\bar{\eta}| = 0.40^{+0.08}_{-0.06}$. Information from $\mathcal{B}(B \rightarrow \rho\gamma)/\mathcal{B}(B \rightarrow K^*\gamma)$ also constrains $|V_{td}/V_{ts}|$. One expects, for example,³⁰ $\mathcal{B}(B^0 \rightarrow \rho^0\gamma) = (0.64 \pm 0.23) \times 10^{-6}$. A recent report³¹ finds $\mathcal{B}(B^0 \rightarrow \rho^0\gamma) < 0.4 \times 10^{-6}$ and $|V_{td}/V_{ts}| < 0.19$ at 90% c.l.

The new CLEO value $f_D = (201 \pm 41 \pm 17)$ MeV³² has an accuracy approaching that of lattice calculations. With increased integrated CLEO luminosity, this quantity will be measured precisely enough to test those calculations, lending weight to their predictions for f_B .

As mentioned, $|V_{td}|$ is quoted with an asymmetric error since it is restricted on the positive side by $B_s - \bar{B}_s$ mixing. This quantity is described by the same diagram as $B^0 - \bar{B}^0$ mixing with the substitution $d \rightarrow s$. We assume $V_{ts} \simeq -V_{cb}$. Then the lower limit $\Delta m_s > 14.5$ ps⁻¹ and the SU(3)-breaking estimate $\xi \equiv f_{B_s} \sqrt{B_{B_s}}/f_B \sqrt{B_B} = 1.21 \pm 0.06$ implies a lower limit on V_{ts}/V_{td} and hence an upper limit on V_{td} . The $(\pm 2\sigma)$ prediction of Ref.⁵ is $\Delta m_s = 17.8^{+15.2}_{-2.7}$ ps⁻¹.

The mixing of B_s and \bar{B}_s can proceed via on-shell shared intermediate states, as described in Fig. 4. One expects (see, e.g.,³³ and references therein) $\Delta\Gamma_s \simeq -\Delta m_s/200$ ($\sim m_b^2/m_t^2$) or $\Delta\Gamma_s/\bar{\Gamma}_s \simeq 0.18(f_{B_s}/200 \text{ MeV})^2$ in lowest order, but a state-of-the-art calculation³⁴ finds $\Delta\Gamma_s/\bar{\Gamma}_s = 0.12 \pm 0.05$ for $f_{B_s} = 245$ MeV. The CDF Collaboration has recently reported a value of $0.65^{+0.25}_{-0.23} \pm 0.01$ ³⁵. Since f_{B_s} is expected to be no larger than about 300 MeV, the CDF result is somewhat larger than anticipated, but not yet in serious conflict with theory.

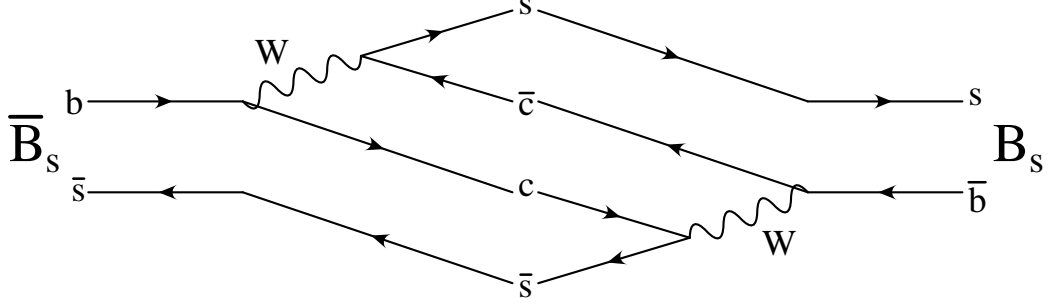


Figure 4: One graph describing B_s - \bar{B}_s mixing, which can correspond to a real intermediate $c\bar{c}s\bar{s}$ state.

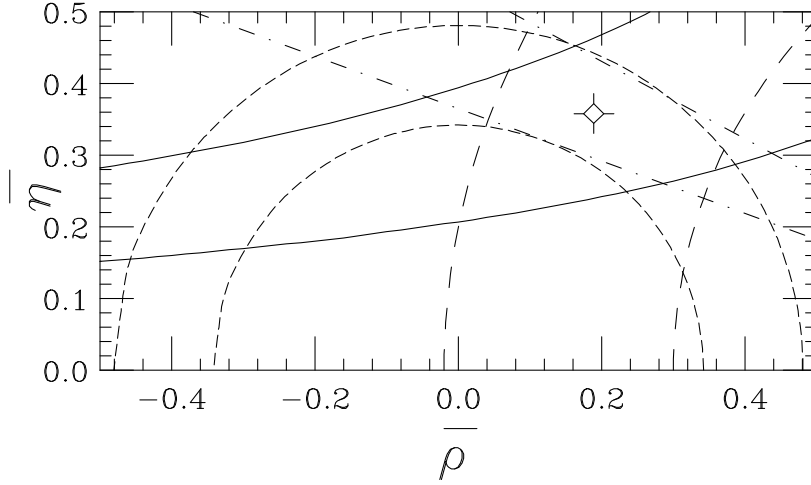


Figure 5: Summary of 95% c.l. constraints on $(\bar{\rho}, \bar{\eta})$ due to $|V_{ub}|$ (short-dashed circles), B^0 - \bar{B}^0 and B_s - \bar{B}_s mixing (dashed circles), CP-violating K^0 - \bar{K}^0 mixing (solid hyperbolae, $\pm 1\sigma$ limits), and $\sin 2\beta$ from CP asymmetry in $B^0 \rightarrow J/\psi K_S$ and related processes (dash-dotted lines).

8 Constraints from K_L , K^+ decays

CP-violating K^0 - \bar{K}^0 mixing is dominated by top quarks in second-order-weak loop diagrams. The parameter ϵ_K describing this mixing depends mainly on $\text{Im}(V_{td}^2)$ and hence measures approximately $\bar{\eta}(1 - \bar{\rho})$, with charmed quarks supplying a small correction. Convenient expressions in Ref. ³⁰ imply that $(\bar{\rho}, \bar{\eta})$ lies between the 1σ boundaries $\bar{\eta}(1.38 - \bar{\rho}) = 0.28$ and $\bar{\eta}(1.26 - \bar{\rho}) = 0.50$. These constraints and those on $(\bar{\rho}, \bar{\eta})$ from $|V_{ub}|$, $|V_{td}|$, and $\sin 2\beta$ select a region around $(0.19^{+0.09}_{-0.07}, 0.36^{+0.05}_{-0.04})$, shown as the plotted point in Fig. 5.

Note the consistency of all these determinations. The main uncertainty is associated with the value of $|1 - \rho - i\eta|$. Direct CP violation in neutral kaon decay (governed by a parameter ϵ'/ϵ) is seen but provides no useful constraint as a result of hadronic uncertainties.

In $K \rightarrow \pi\nu\bar{\nu}$ decays, higher-order weak diagrams govern the weak quark transition $s \rightarrow d\nu\bar{\nu}$. For $K^+ \rightarrow \pi^+\nu\bar{\nu}$ the top quark in the loop dominates but there is also a charm contribution, so that the rate measures $|1.3 - \bar{\rho} - i\bar{\eta}|$. An experiment (E787, E949) at Brookhaven sees three events ³⁶, corresponding to a branching ratio $\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) = (1.47^{+1.30}_{-0.82}) \times 10^{-10}$. Comparing with the Standard Model prediction ³⁷ of $(0.78 \pm 0.12) \times 10^{-10}$, the result is still consistent with a large $(\bar{\rho}, \bar{\eta})$ region. Proposals for Fermilab and JPARC seek a sample of 100 events which could determine $|1.3 - \bar{\rho} - i\bar{\eta}|$ to 5%.

The process $K_L \rightarrow \pi^0\nu\bar{\nu}$ is purely CP-violating and measures $\bar{\eta}^2$. One expects $\mathcal{B} = (3.0 \pm 0.6) \times 10^{-11}$ ³⁷. An experiment at KEK (PS E391) has taken data whose single-event sensitivity

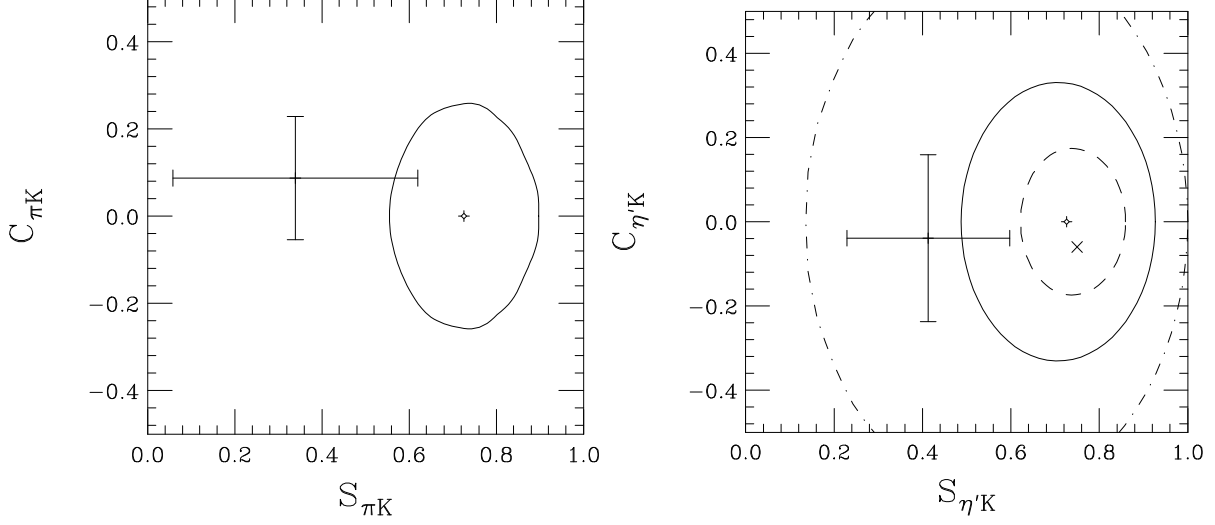


Figure 6: Left: $B^0 \rightarrow \pi^0 K_S$; solid ellipse: allowed region boundary. Right: $B^0 \rightarrow \eta' K_S$; solid curve: bounds based on flavor SU(3) and measurements of other processes; dot-dashed curve: bounds prior to latest set of such measurements; dashed curve: neglecting processes involving spectator quark.

should be an order of magnitude above the Standard Model value¹⁵; this represents tremendous progress in the past few years. The eventual goal of proposed experiments at JPARC and Brookhaven is to be sensitive at the Standard Model level.

9 V_{ts} and V_{tb} : $b \rightarrow s\gamma$, top quark decays, unitarity

One may obtain a lower limit on $|V_{tb}^* V_{ts}|$ from $B_s - \bar{B}_s$ mixing, the assumption that $\xi = 1.2 \pm 0.1$, and $B_d - \bar{B}_d$ mixing, yielding³⁰ $|V_{tb}^* V_{ts}| > 0.034$. An upper limit can be extracted from the top quark contribution to $b \rightarrow s\gamma$ ³⁸: $V_{tb}^* V_{ts} = -0.047 \pm 0.008$, which has the expected sign.

The fact that top quark decays $t \rightarrow b\ell^+\nu_\ell$ dominate over those with no b has been used by the CDF Collaboration to conclude on the basis of Run I data that $|V_{tb}|^2/(|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2) = 0.94^{+0.31}_{-0.24}$. A slightly less stringent limit has been placed for a sample of 108 pb^{-1} of Run II data: $0.54^{+0.49}_{-0.39}$. The assumption that CKM matrix is 3×3 is crucial to interpretation of these results as a useful bound on $|V_{tb}|^2$. The lower bounds on magnitudes of the third row of the CKM matrix are almost non-existent if there are more than 3 families⁴.

10 Processes giving β

Many experiments measure a time-dependent CP asymmetry in B decays of the form $A(t) = -C \cos(\Delta mt) + S \sin(\Delta mt)$. Results involving the subprocess $b \rightarrow c\bar{c}s$, such as $B^0 \rightarrow J/\psi K_S$, show beautiful agreement with Standard Model fits, as illustrated in Fig. 5.

For processes dominated by the $b \rightarrow s$ penguin amplitudes one expects $C = 0$, $S = \sin(2\beta) = 0.74 \pm 0.05$. One can estimate contributions from other amplitudes using new measurements and flavor SU(3), as in the processes $B^0 \rightarrow \pi^0 K_S$ ³⁹ and $B^0 \rightarrow \eta' K_S$ ⁴⁰. We show in Fig. 6 examples of bounds on allowed deviations from the Standard Model predictions. We have applied a scale factor in averaging $\eta' K_S$ results from BaBar⁴¹ ($S_{\eta' K_S} = 0.27 \pm 0.14 \pm 0.03$, $C_{\eta' K_S} = -0.21 \pm 0.10 \pm 0.03$) and Belle⁴² ($S_{\eta' K_S} = 0.65 \pm 0.18 \pm 0.04$, $C_{\eta' K_S} = 0.19 \pm 0.11 \pm 0.05$), so the plotted data points have larger error bars than quoted in, e.g., Ref.³. The corresponding values for $\pi^0 K_S$ are $S_{\pi^0 K_S} = 0.35^{+0.30}_{-0.33} \pm 0.04$, $C_{\pi^0 K_S} = -0.21 \pm 0.10 \pm 0.03$ (BaBar⁴¹) $S_{\pi^0 K_S} = 0.30 \pm 0.59 \pm 0.11$, $C_{\pi^0 K_S} = 0.12 \pm 0.20 \pm 0.07$ (Belle⁴²). There is not, in my opinion, evidence yet

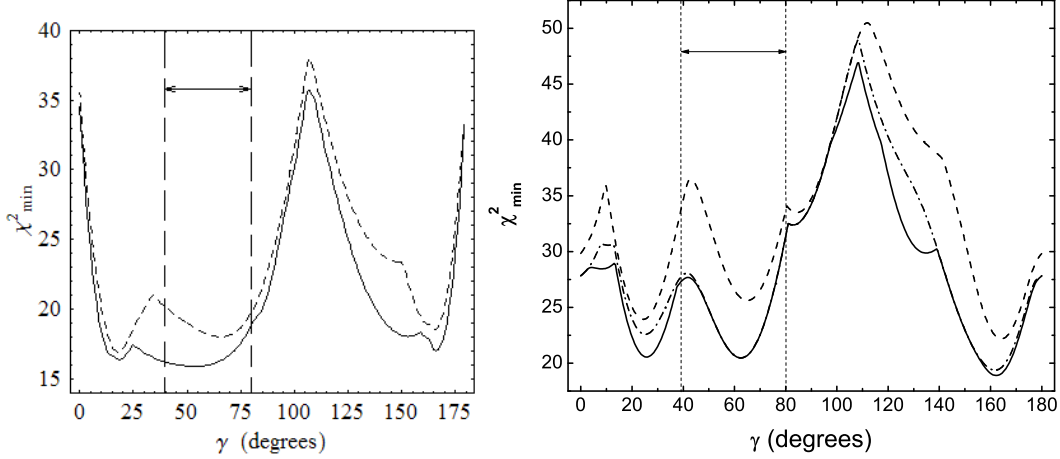


Figure 7: Results of χ^2 fits to $B \rightarrow PP$ (left) and $B \rightarrow VP$ (right) decays. In PP fit the lower curve corresponds to a fit with two extra parameters. In VP fit the solid curve corresponds to a fit with no constraint between two independent penguin amplitudes, while the dot-dashed and dashed curves correspond respectively to constraining these amplitudes to be relatively real, and equal and opposite.

for substantial deviations from the Standard Model predictions, but the situation bears watching both in the processes illustrated in Fig. 6 and in the decay $B^0 \rightarrow \phi K_S$, also dominated by the $b \rightarrow s$ penguin. With improved data one could well have evidence for new physics.

11 $B^0 \rightarrow \pi^+\pi^-, \pi^\pm\rho^\mp, \rho^+\rho^-$ and α

The time-dependent parameters S in the processes $B^0 \rightarrow \pi^+\pi^-$, $B^0 \rightarrow \rho^\pm\pi^\mp$, and $B^0 \rightarrow \rho^+\rho^-$ would measure just $\sin 2\alpha$ if one could neglect the effect of penguin “pollution” of the dominant tree amplitudes. In $B \rightarrow \pi\pi$ the penguin/tree amplitude ratio was estimated to be $\sim 70\%$ from $B \rightarrow K\pi$ decays⁴³, leading to the estimate $\alpha = (103 \pm 17)^\circ$. In $B^0 \rightarrow \rho^\pm\pi^\mp$ the relative penguin contribution is found to be less⁴⁴, leading to $\alpha = (95 \pm 16)^\circ$, while in $B^0 \rightarrow \rho^+\rho^-$ a recent BaBar measurement, combined with an estimate of penguin effects, gives⁴⁵ $\alpha = (96 \pm 10 \pm 4 \pm 13)^\circ$.

12 $B \rightarrow PP, VP$ and γ

One can perform fits to rates and CP asymmetries in flavor SU(3) based on amplitudes designated by T, P, C, \dots (denoting tree, penguin, color-suppressed, etc.), for $B \rightarrow PP$ ⁴⁶ and $B \rightarrow VP$ ⁴⁷ decays, where P, V denote light (pseudoscalar, vector) mesons composed of u, d, s quarks. The $B \rightarrow PP$ fits involve 26 observables while the $B \rightarrow VP$ fits involve 34. The results are shown in Fig. 7. We place greatest reliance on the fits described by the solid curves, corresponding to $\chi^2_{\min}/\text{d.f.} = 16.0/13$ (left) and $\chi^2_{\min}/\text{d.f.} = 20.5/22$ (right). The fit to PP decays gives a rather shallow minimum around $\gamma = 54^\circ$ whose stability we do not yet trust, while the fit to VP decays gives $\gamma = (63 \pm 6)^\circ$. The combined fit gives local minima at $\gamma \simeq 55^\circ$ and 61° after being updated in view of new data presented at ICHEP 04⁴⁸.

13 Beyond the 3×3 CKM matrix

Does a fourth family of quarks and leptons exist? Any fourth neutrino must be heavy; only 3 light neutrinos are seen in Z decay⁴⁹. A direct search for b' heavier than Z (looking for $b' \rightarrow bZ$) by CDF at the Tevatron⁵⁰ excludes $100 < m(b') < 199 \text{ GeV}/c^2$. However, looking outside the familiar pattern, existing quark-lepton families belong to 16-dimensional multiplets of the grand

unified group $SO(10)$, consisting of $\mathbf{1} + \mathbf{5}^* + \mathbf{10}$ of $SU(5)$. The smallest representation $\mathbf{27}$ of E_6 , an interesting group containing $SO(10)$, is $\mathbf{16} + \mathbf{10} + \mathbf{1}$ of $SO(10)$. The $\mathbf{10}$ of $SO(10)$ consists of isosinglet quarks “ h, \bar{h} ” with charge $Q = \pm 1/3$, and isodoublet leptons. The $SO(10)$ singlets are candidates for sterile neutrinos, one for each family.

The exotic h quarks can mix with b and push its mass down with respect to t ⁵¹. Production signatures of $h\bar{h}$ at the Tevatron and LHC have been investigated⁵². Through decays of h to $Z + b$, $W + t$, and possibly Higgs + b , one can reach h masses 270–320 GeV/ c^2 at the Tevatron.

14 Summary

- (1) The CKM unitarity relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ seems valid, though questions remain.
- (2) Improved V_{cd} and V_{cs} values still are consistent with $V_{cd} = -V_{us}$ and $V_{cs} = V_{ud}$.
- (3) Errors on $V_{cb} \sim 42 \times 10^{-3}$ from inclusive analyses are being reduced; a 1σ error of about 3% is a conservative guess. Lattice QCD will help exclusive analyses catch up.
- (4) Lower V_{ub} values $[(3.26 \pm 0.62) \times 10^{-3}]$ are obtained in exclusive $b \rightarrow u$ decays than those $[(4.66 \pm 0.43) \times 10^{-3}]$ in inclusive decays. One needs a better understanding of form factors and quark-hadron duality. An average $\sim 4.2 \times 10^{-3}$ is known at present to $\simeq 15\%$.
- (5) Errors in $|V_{td}| \simeq (8.3_{-1.8}^{+1.2}) \times 10^{-3}$ (95% c.l.) are due mainly to lattice errors in $f_B\sqrt{B_B}$. One expects a major improvement from detection of $B_s - \bar{B}_s$ mixing.
- (6) The results of an $SU(3)$ fit giving $\gamma = (63 \pm 6)^\circ$ are being validated by the most recent measurements of rare B decay branching ratios⁴⁸. The detection of $b \rightarrow d$ penguins at expected rates⁴² has provided partial confirmation.
- (7) There is no evidence against $V_{ts} \simeq -V_{cb}$, $V_{tb} \simeq 1$.
- (8) It’s time for a theory of quark masses and CKM elements!

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